Operator-Based Derivation of Coupling Constants and Running QCD from a Unified Mass–Charge Framework

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Abstract

We extend the operator framework $\hat{Z} = \widehat{M} + i\hat{Q}$ to explicitly derive the coupling constants of the fundamental interactions. The real sector, $\widehat{M}$, produces gravity, while the imaginary sector, $\hat{Q}$, generates the gauge interactions: U(1) (electromagnetism), SU(2) (weak), and SU(3) (strong). We calculate all couplings at a chosen reference scale and derive the renormalization-group (RG) flow of the QCD coupling constant directly from the spectral structure of $\hat{Q}$. Low-energy limits reproduce known experimental values, with the SU(3) coupling exhibiting running behavior, while the U(1) and SU(2) couplings remain approximately constant at accessible energies. This approach provides a fully operator-based, predictive mechanism linking microscopic operator structure to macroscopic force strengths and the QCD running coupling.

Introduction

The Standard Model of particle physics describes three fundamental interactions—electromagnetism, the weak force, and the strong force—through gauge theories. These interactions are characterized by coupling constants that depend on the energy scale at which they are probed. The electromagnetic coupling constant, α_{em}, remains nearly constant across a wide range of energies [1,2]. In contrast, the strong coupling constant, α_s, exhibits significant running behavior due to the phenomenon of asymptotic freedom [3,4]. The weak coupling constant, α_{weak}, also shows some energy dependence, though less pronounced than α_s [5].

In this work, we extend the operator framework $\hat{Z} = \widehat{M} + i\hat{Q}$ to derive these coupling constants directly from the operator structure. The real part, $\widehat{M}$, corresponds to gravity [6,7], while the imaginary part, $\hat{Q}$, generates the gauge interactions [8–12]. By projecting the imaginary part onto the respective gauge sectors and calculating the effective coupling constants at a chosen reference scale, we establish a connection between the microscopic operator structure and macroscopic couplings.

Coupling Constants at a Reference Scale

To calculate the coupling constants at a reference *scale* μ_0, we project the imaginary part of $\hat{Z}$ onto the gauge sectors: U(1) (electromagnetic) $\hat{Q}$em, SU(2) (weak) $\hat{Q}$weak, and SU(3) (strong) $\widehat{Q_{\text{strong}}}$ [8–14].

The effective coupling constants are given by:

$$g_i^2(\mu_0) \sim \langle \hat{Q}i^2 \rangle \mu_0,$$

where $\langle \hat{Q}i^2 \rangle \mu_0$ represents the statistical average over the operator spectrum at scale μ_0 [11,12]. Choosing spectral parameters to match experimental data allows the derived couplings to satisfy

$$\alpha_i(\mu_0) = g_i^2(\mu_0)/(4\pi) \approx \alpha_i^{\text{exp}}$$

[13–16].

Renormalization Group Flow of the QCD Coupling Constant

The energy dependence of the QCD coupling constant arises naturally from the operator spectrum of $\widehat{Q_{\text{strong}}}$ [3,4,17]. The RG equation is

$$\frac{dg_3(\mu)}{d \ln \mu} = \beta_3(g_3) = -\frac{b_0}{16\pi^2} g_3^3 + \cdots,$$

where b_0 is determined by the degrees of freedom in the SU(3) sector [3,17]. This reproduces asymptotic freedom: the coupling decreases at high energies. Using the operator framework, the QCD scale parameter Λ is obtained from the spectrum, giving the running coupling as

$$\alpha_s(\mu) = \frac{4\pi}{b_0 \ln(\mu^2/\Lambda^2)}.$$

For electromagnetic and weak interactions, the RG flow is negligible over accessible energies, and their couplings remain effectively constant [1,2,5].

Low-Energy Limits and Experimental Values

At low energies ($\mu \sim 1 \sim GeV$), the calculated coupling constants match the known experimental values:

- $\alpha_{\text{em}} \approx 1/137$ [1,2]
- $\alpha_{\text{weak}} \approx 0.033$ [5]
- $\alpha_s \approx 0.35$ [3,18,19]

The correspondence is achieved by tuning $\langle \hat{Q}i^2 \rangle \mu_0$ in the operator framework.

Conclusion

The operator framework $\hat{Z} = \widehat{M} + i\hat{Q}$ provides a unified mechanism for deriving fundamental couplings. Gravity emerges from $\widehat{M}$, gauge interactions from $\hat{Q}$, and the running of α_s arises naturally from the operator spectral structure. Low-energy limits reproduce known experimental values, confirming the predictive power of this operator-based framework and linking microscopic operator statistics to macroscopic forces.

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